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**CERTIFIED PUBLIC ACCOUNTANT  
FOUNDATION 1 EXAMINATION**

**F1.1: BUSINESS MATHEMATICS AND QUANTITATIVE  
METHODS**

**TUESDAY: 4 JUNE 2019**

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**INSTRUCTIONS:**

1. **Time Allowed: 3 hours 15 minutes** (15 minutes reading and 3 hours writing).
2. This examination has **seven** questions and only **five** questions are to be attempted.
3. Marks allocated to each question are shown at the end of the question.
4. Show all your workings, where applicable.

## QUESTION ONE

- (a) Explain the term 'amortisation schedule'. (2 Marks)
- (b) Minani, a secondary school teacher at GS Gasogi earns a monthly salary of Frw 250,000. He has applied for a loan from a bank which offers a maximum amount of fifteen times a teacher's monthly salary at an interest rate of 17% per annum compound interest. The loan is amortised by equal annual repayments over the next five years.

### REQUIRED:

- (i) Compute the maximum loan amount Minani can apply for. (2 Marks)
- (ii) Determine the value of each payment. (4 Marks)
- (iii) Construct an amortisation for the repayment of the loan. (4 Marks)
- (c) Kamembe Tea Factory will have to spend Frw 12.5 million to expand the current factory to modern standards in a period of three years. Current investment rates are at 5.5% per annum compound interest.

### REQUIRED:

Compute the:

- (i) Amount that should be invested, if interest is compounded on a 4 monthly period. (4 Marks)
- (ii) Actual percentage rate. (4 Marks)
- (Total 20 Marks)**

## QUESTION TWO

- (a) Distinguish between grouped and ungrouped frequency distribution. (2 Marks)
- (b) Radiant Insurance Company in Kigali issued thousands of insurance policies between 2015 and 2017 as shown in the table below.

	2015	2016	2017
Motor vehicle	48	44	42
Health	40	37	43
Property	31	35	41
Fire	13	18	20

### REQUIRED:

- (i) Represent the above data on a compound bar chart. (5 Marks)
- (ii) Identify the best performing policy, giving a reason for your choice. (2 Marks)
- (c) Kicukiro Stores deal in television sets and their accessories. The management of Kicukiro Stores have collected the following data about the age distribution of Nyarugenge residents as a basis for formulating their marketing strategy.

Age (x)	No of people (f)
20-25	1,800
25-30	2,500
30-35	2,200
35-40	2,700
40-45	2,000
45-50	1,500

**REQUIRED:**

Compute the:

- (i) mean age of the residents. **(4 Marks)**
- (ii) standard deviation of the age of residents. **(5 Marks)**
- (iii) coefficient of variation. **(2 Marks)**

**(Total 20 Marks)**

**QUESTION THREE**

- (a) Explain the term 'pooled variance' in statistics. **(2 Marks)**

- (b) Clement Hirwa deals in three types of shoes whose selling prices are Frw 10,000, Frw 15,000 and Frw 20,000 for each type respectively. Pierre Twagirimana, Hirwa's salesperson gets a 5% commission on every type of shoe that he sells.

Hirwa believes that there is a 30% chance that a customer will purchase a Frw 10,000 shoe, a 20% chance that a customer will purchase a Frw 15,000 shoe and a 10% chance that a customer will purchase a Frw 20,000 shoe.

The probability that a customer will not buy a shoe is  $\frac{2}{5}$ . It is also known that no customer will buy more than two pairs of shoes.

**Hint:** Let the random variable X denote Twagirimana's potential commission.

**REQUIRED:**

- (i) Construct a probability distribution for the random variable X. **(2 Marks)**
- (ii) Calculate Twagirimana's expected commission. **(2 Marks)**
- (c) Mgahinga Ltd decided to downsize its labour force due to persistent decline in sales. As a result, in a random sample of 100 young employees 35 were laid off and in an independent random sample of 100 old employees 30 were also laid off.

**REQUIRED:**

- (i) Find a 99 % confidence interval if the difference of proportions of the young and old were equal. **(6 Marks)**
- (ii) Comment on your results. **(1 Mark)**
- (d) In Nyakabanda it is claimed that the type of phone held by individuals relates to the income the individual gets. To dispel this claim a sample of 500 individuals were asked the type of phones they held and their income status.

The results were recorded in the following contingency table.

	Type of phone (Tech series)			
Income status	M Tech	N Tech	P Tech	Total
Low income	143	70	37	250
Medium income	90	67	43	200
High income	17	13	20	50

**REQUIRED:**

Using Chi-square method, test whether ownership of phone type is independent of income status at 5% level of significance.

**(7 Marks)**  
**(Total 20 Marks)**

#### QUESTION FOUR

- (a) Explain the term 'value index'. **(2 Marks)**
- (b) Gatera & Brothers shop in Gitarama town imports spare parts for Carinae cars (x) and motor cycles (y). Their yearly sales records (in thousands) were kept for the period 2008 and 2017 as shown in the following table:

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
x	42	50	36	55	52	48	58	55	45	60
y	73	75	61	67	80	71	73	85	66	79

#### REQUIRED:

- (i) Plot and identify the type of correlation in the diagram for the above data. **(4 Marks)**
- (ii) Compute the regression equation by the least squares method. **(11 Marks)**
- (iii) Using the regression equation, estimate the carinae car spare parts that were sold when 85,000 spare parts for motor cycles were sold in 2018. **(3 Marks)**
- (Total 20 Marks)**

## QUESTION FIVE

- (a) Explain the **three** main uses of the coefficient of correlation. **(3 Marks)**
- (b) The following table shows the percentage of degrees awarded to women in the disciplines which were traditionally dominated by men in 2005 and 2017.

Discipline	2005	2017
Dentistry	2	11.9
Law	11.5	28.5
Medicine	11.2	23.1
Optometry	4.2	13
Orthopedic	2.8	15.7
Radiography	1.1	7.2
Theology	5.5	13.1
Veterinary medicine	11.2	28.9

### REQUIRED:

- (i) Determine the correlation between the percentage of degree awarded to women in 2005 and 2017 by Kendall's method. **(8 Marks)**
- (ii) Comment on the correlation coefficient. **(1 Mark)**
- (c) The following table shows precedence activities in a fast simple project of preparing tea at break time at a Kindergarten.

Activity	Activity description	Preceding activity	Duration (seconds)
A	Fill electric kettle with water	-	8
B	Boil the water	A	60
C	Find and clean teapot and cups	-	25
D	Put tea leaves in teapot	C	6
E	Pour water in the teapot	B, D	10
F	Allow tea to brew	E	45
G	Put milk in the cups	C	10
H	Put tea into the cups	F, G	20

### REQUIRED

- (i) Draw an activity network for the above project. **(3 Marks)**
- (ii) Identify the earliest start time (EST) for each activity. **(3 Marks)**
- (iii) Determine the critical path and duration of the project. **(2 Marks)**
- (Total 20 Marks)**

## QUESTION SIX

- (a) Identify the main use of linear programming in business applications. **(2 Marks)**
- (b) Smoking in Huye town is highly restricted, at present. Rukundo Hotel has a smoking section (S) and a non-smoking section (N). On average a smoker spends Frw 15,000 per visit to the hotel while a non-smoker spends Frw 10,000 per visit.

In an interview on one of the radio stations, the hotel manager reechoed that the hotel has two designated sections, that is to say, for smokers and for non-smokers. He further informed that if a client who smokes visits the hotel and the smoking section is open the hotel makes Frw 15,000. However, if only the non-smoking section is open, the customer leaves. On the other hand if a non-smoker visits and finds that it is only the smoking section that is open, he leaves, while if he finds the non-smoking section open the hotel makes Frw 10,000.

### REQUIRED:

- (i) Construct a payoff matrix for the manager of Rukundo Hotel. **(1 Mark)**
- (ii) Explain whether there is a saddle point in this game **(2 Mark)**

Determine the probability that a customer the manager should expect will be a:

- (iii) Smoker. **(4 Marks)**
- (iv) Non-smoker. **(1 Mark)**
- (c) Beds World Ltd (BWL) manufactures two types of beds: deluxe adjustable and deluxe ordinary. Each bed has to be processed by three machines and the time taken on each machine is indicated in the table below.

	Maximum time Available (hours)	Time (hours)	
		Deluxe adjustable	Deluxe ordinary
Machine 1	36	6	2
Machine 2	30	3	5
Machine 3	20	1	4

BWL realises a profit of Frw 250,000 on a deluxe adjustable bed and Frw 200,000 on a deluxe ordinary bed.

### REQUIRED:

- (i) Express the above information as linear programming model. **(2 Marks)**
- (ii) Using the simplex tableau method, find the number of each type of bed BWL should produce in order to maximise profit. **(8 Marks)**
- (Total 20 Marks)**

## QUESTION SEVEN

(a) Outline any **one** limitation, in decision theory, for the following criteria:

- (i) Maximin. **(1 Mark)**
- (ii) Maximax. **(1 Mark)**

(b) Nkunzi a youth group in Kirehe district is in the process of choosing between a mobile money project and a supermarket. The expected profits are as in the table below:

	Profit (strong demand)	Profit/loss (weak demand)
Mobile money	Frw 125,000	Frw 75,000
Supermarket	Frw 300,000	Frw (187,500)
Probability of demand	0.7	0.3

### REQUIRED:

- (i) Advise Nkunzi youth group based on expected monetary value approach on the decision to take under conditions of risk. **(5 Marks)**
- (ii) Determine the expected value of perfect information. **(3 Marks)**
- (c) Television Network in Kigali is planning to produce television series programs next season. The first television series program is a detective show with a probability  $\frac{3}{10}$  and expected to earn profit of Frw 300,000 and a probability of  $\frac{7}{10}$  with expected loss of Frw 80,000.

The second possible television series program is a comedy show with a probability of  $\frac{2}{5}$  and expected to earn a profit of Frw 200,000 and a probability of  $\frac{3}{5}$  and expected loss of Frw 100,000.

If a comedy television series program is successful the television network has the option of developing a second spin-off television series using one of the stars from the first series. The spin-off would yield a profit of Frw 80,000 with a probability of  $\frac{1}{5}$  or a loss of Frw 40,000 with a probability of  $\frac{4}{5}$ .

### REQUIRED:

- (i) Construct a decision tree to illustrate the above information. **(6 Marks)**
  - (ii) Determine the optimal action. **(4 Marks)**
- (Total 20 Marks)**

## FORMULAE

1. Profit = Total Revenue – Total Cost
2. Spearman's Rank correlation coefficient  $r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$
3. Harmonic mean (grouped data)  $hm = \frac{n}{\sum \frac{f}{x}}$
4. Mean  $\bar{x} = A + \frac{\sum fd}{\sum f}$  or Mean  $\bar{x} = \frac{\sum fx}{\sum f}$
5. Geometric mean  $GM = \text{Anti log } \frac{1}{n} \sum \log_{10} x$
6. Median  $= Lb + \left( \frac{\frac{N}{2} - Cfb}{fm} \right) C$
7. Mode  $= lm + \left( \frac{d_1}{d_1 + d_2} \right) C$
8. Co-efficient of variation  $= \frac{\text{Standard deviation}}{\text{Mean}} \times 100$
9. Standard deviation  $\delta = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$
10. Least squares regression equation of y on x is given by  $y = a + bx$   
Where;  $b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$  and  $a = \frac{\sum y}{n} - \frac{b\sum x}{n}$
11. The coefficient of correlation r =  $\frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2} \times \sqrt{n\sum y^2 - (\sum y)^2}}$
12. Standardizing normal  $z = \frac{\bar{x} - \mu}{\sigma}$
13. Confidence interval for a small sample  $= \bar{x} \pm t_{\alpha} \frac{s}{\sqrt{n-1}}$
14. Confidence interval for a population  $= \bar{x} \pm z_{\frac{\alpha}{2}} \frac{\delta}{\sqrt{n}}$
- Confidence interval for difference between proportions
15. Lower limit  $= (p_1 - p_2) - z_{\frac{\alpha}{2}} \sqrt{\frac{P_1q_1}{n} + \frac{P_2q_2}{n}}$
16. Confidence interval for difference between proportions

$$\text{Upper limit} = (p_1 - p_2) + z_{\frac{\alpha}{2}} \sqrt{\frac{p_1 q_1}{n} + \frac{p_2 q_2}{n}}$$

17. The Chi-square  $X^2 = \sum \frac{(O - E)^2}{E}$  where O = Observed value and E = Expected value

18. Pearson coefficient of skewness  $Sk = \frac{(\bar{x} - \text{mode})}{s_d}$  or  $Sk = \frac{3(\bar{x} - \text{median})}{s_d}$

19. Paasche's price index =  $\frac{\sum (p_1 \times q_1)}{\sum (p_0 \times q_1)} \times 100$

20. Value index number =  $\frac{\sum (p_1 \times q_1)}{\sum (p_0 \times q_0)} \times 100$

21. Conditional probability  $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

22. Expectation  $E(X) = \sum xP(X = x)$

23.  $P = \frac{A}{(1 + r)^n}$  where A is the compound loan amount at the end of the period, P is the

Principle, r is the rate and n is the number of period.

24.  $FV = P \frac{(1 + r)^n - 1}{r}$ , where FV is the future value of an annuity, P is the period payment, r = rate of period and n is the number of period.

25.  $R = \frac{Pr}{\left(1 - \frac{1}{(1 + r)^n}\right)}$ , where P = principle money borrowed, n = number of years required to repay the loan, r = the compound interest rate charged on the loan, R = value of each annual repayment.

26. Actual percentage rate =  $\left(1 + \frac{r}{n}\right)^n - 1$ , where n is the period in years and r is the annual investment rates (compound interest)

27. Expected value of perfect information (VPI) = Expected monetary value (EMV) under certainty – EMV under risk

CUMULATIVE NORMAL DISTRIBUTION  $P(z)$

CUMULATIVE NORMAL DISTRIBUTION $P(z)$											ADD								
Z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	0.0000	0040	0080	0120	0160	0199	0239	0279	0319	0359	4	8	12	16	20	24	28	32	36
0.1	0.0398	0438	0478	0517	0557	0596	0636	0675	0714	0753	4	8	12	16	20	24	28	32	36
0.2	0.0793	0832	0871	0910	0948	0987	1026	1064	1103	1141	4	8	12	15	19	22	27	31	35
0.3	0.1179	1217	1255	1293	1331	1368	1406	1443	1480	1517	4	8	11	15	19	22	26	30	34
0.4	0.1554	1591	1628	1664	1700	1736	1772	1808	1844	1879	4	7	11	14	18	22	25	29	32
0.5	0.1915	1950	1985	2019	2054	2088	2123	2157	2190	2224	3	7	10	14	17	21	24	27	31
0.6	0.2257	2291	2324	2357	2389	2422	2454	2486	2517	2549	3	6	10	13	16	19	23	26	29
0.7	0.2580	2611	2642	2673				3	6	9	3	6	9	12	15	19	22	25	28
					2704	2734	2764	2794	2823	2852	3	6	9	12	15	18	21	24	27
0.8	0.2881	2910	2939	2967	2995	3023		3	6	8	3	6	8	11	14	17	20	22	25
							3051	3078	3106	3133	3	5	8	11	13	16	19	22	24
0.9	0.3159	3186	3212	3238	3264	3289		3	5	8	3	5	8	10	13	16	18	21	23
						3315		3340	3365	3389	2	5	7	10	12	15	17	20	22
1.0	0.3413	3438	3461	3485	3508			2	5	7	2	5	7	10	12	14	17	19	22
						3531	3554	3577	3599	3621	2	4	7	9	11	13	15	18	20
1.1	0.3643	3665	3686	3708				2	4	6	2	4	6	8	11	13	15	17	19
					3729	3749	3770	3790	3810	3830	2	4	6	8	10	12	14	16	18
1.2	0.3849	3869	3888	3907	3925			2	4	6	2	4	6	8	10	11	13	15	17
						3944	3962	3980	3997	4015	2	4	5	7	9	11	13	14	16
1.3	0.4032	4049	4066	4082	4099	4115	4131	4147	4162	4177	2	3	5	6	8	10	11	13	14
1.4	0.4192	4207	4222	4236	4251	4265	4279	4292	4306	4319	1	3	4	6	7	8	10	11	13
1.5	0.4332	4345	4357	4370	4382	4394	4406	4418	4429	4441	1	2	4	5	6	7	8	10	11
1.6	0.4452	4463	4474	4484	4495	4505	4515	4525	4535	4545	1	2	3	4	5	6	7	8	9
1.7	0.4554	4564	4573	4582	4591	4599	4608	4616	4625	4633	1	2	3	3	4	5	6	7	8
1.8	0.4641	4649	4656	4664	4671	4678	4686	4693	4699	4706	1	1	2	3	4	4	5	6	6
1.9	0.4713	4719	4726	4732	4738	4744	4750	4756	4761	4767	1	1	2	2	3	4	4	5	5
2.0	0.4772	4778	4783	4788	4793	4798	4803	4808	4812	4817	0	1	1	2	2	3	3	4	4
2.1	0.4821	4826	4830	4834	4838	4842	4846	4850	4854	4857	0	1	1	2	2	2	3	3	4
2.2	0.4861	4864	4868	4871	4875	4878	4881	4884	4887	4890	0	1	1	1	2	2	2	3	3
2.3	0.4893	4896	4898	4901	4904	4906	4909	4911	4913	4916	0	0	1	1	1	2	2	2	2
2.4	0.4918	4920	4922	4925	4927	4929	4931	4932	4934	4936	0	0	1	1	1	1	1	2	2
2.5	0.4938	4940	4941	4943	4945	4946	4948	4949	4951	4952									
2.6	0.4953	4955	4956	4957	4959	4960	4961	4962	4963	4964									
2.7	0.4965	4966	4967	4968	4969	4970	4971	4972	4973	4974									
2.8	0.4974	4975	4976	4977	4977	4978	4979	4979	4980	4981									
2.9	0.4981	4982	4982	4983	4984	4984	4985	4985	4986	4986									
3.0	0.4987	4990	4993	4995	4997	4998	4998	4999	4999	5000									

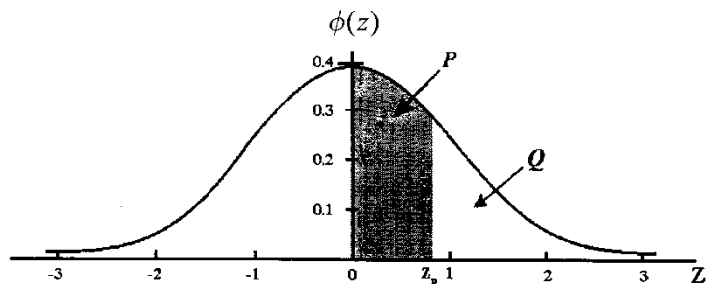
The table gives  $P(z) = \int_0^z \phi(z) dz$

If the random variable Z is distributed as the standard normal distribution  $N(0,1)$  then:

1.  $P(0 < Z < z_p) = P(\text{Shaded Area})$

2.  $P(Z > z_p) = Q = \frac{1}{2} - P$

3.  $P(Z > |z_p|) = 1 - 2P = 2Q$



**PERCENTAGE POINTS OF THE CHI-SQUARE ( $\chi^2$ ) DISTRIBUTION  $\chi^2_Q$**

Probability Q										
$\nu$	0.995	0.990	0.975	0.950	0.100	0.050	0.025	0.010	0.005	0.001
1	0.004393	0.004577	0.004982	0.005393	2.706	3.841	5.024	6.635	7.879	10.83
2	0.01000	0.02010	0.05060	0.10260	4.605	5.991	7.378	9.210	10.60	13.82
3	0.07171	0.11480	0.21580	0.35180	6.251	7.815	9.348	11.34	12.84	16.27
4	0.20700	0.29710	0.48440	0.71070	7.779	9.488	11.14	13.28	14.86	18.47
5	0.41170	0.55430	0.83120	1.14500	9.236	11.07	12.83	15.09	16.75	20.52
6	0.67570	0.87210	1.23700	1.63500	10.64	12.59	14.45	16.81	18.55	22.46
7	0.98930	1.23900	1.69000	2.16700	12.02	14.07	16.01	18.48	20.28	24.32
8	1.34400	1.64600	2.18000	2.73300	13.36	15.51	17.53	20.09	21.95	26.12
9	1.73500	2.08800	2.70000	3.32500	14.68	16.92	19.02	21.67	23.59	27.88
10	2.15600	2.55800	3.24700	3.94000	15.99	18.31	20.48	23.21	25.19	29.59
11	2.60300	3.05300	3.81600	4.57500	17.28	19.68	21.92	24.73	26.76	31.26
12	3.07400	3.57100	4.40400	5.22600	18.55	21.03	23.34	26.22	28.30	32.91
13	3.56500	4.10700	5.00900	5.89200	19.81	22.36	24.74	27.69	29.82	34.53
14	4.07500	4.66000	5.62900	6.57100	21.06	23.68	26.12	29.14	31.32	36.12
15	4.60100	5.22900	6.26200	7.26100	22.31	25.00	27.49	30.58	32.80	37.70
16	5.14200	5.81200	6.90800	7.96200	23.54	26.30	28.85	32.00	34.27	39.25
17	5.69700	6.40800	7.56400	8.67200	24.77	27.59	30.19	33.41	35.72	40.79
18	6.26500	7.01500	8.23100	9.39000	25.99	28.87	31.53	34.81	37.16	42.31
19	6.84400	7.63300	8.90700	10.12000	27.20	30.14	32.85	36.19	38.58	43.82
20	7.43400	8.26000	9.59100	10.85000	28.41	31.41	34.17	37.57	40.00	45.31
21	8.03400	8.89700	10.28000	11.59000	29.62	32.67	35.48	38.93	41.40	46.80
22	8.64300	9.54200	10.98000	12.34000	30.81	33.92	36.78	40.29	42.80	48.27
23	9.26000	10.20000	11.69000	13.09000	32.01	35.17	38.08	41.64	44.18	49.73
24	9.88600	10.86000	12.40000	13.85000	33.20	36.42	39.36	42.98	45.56	51.18
25	10.52000	11.52000	13.12000	14.61000	34.38	37.65	40.65	44.31	46.93	52.62
26	11.16000	12.20000	13.84000	15.38000	35.56	38.89	41.92	45.64	48.29	54.05
27	11.81000	12.88000	14.57000	16.15000	36.74	40.11	43.19	46.96	49.64	55.48
28	12.46000	13.56000	15.31000	16.93000	37.92	41.34	44.46	48.28	50.99	56.89
29	13.12000	14.26000	16.05000	17.71000	39.09	42.56	45.72	49.59	52.34	58.30
30	13.79000	14.95000	16.79000	18.49000	40.26	43.77	46.98	50.89	53.67	59.70
40	20.71000	22.16000	24.43000	26.51000	51.81	55.76	59.34	63.69	66.77	73.40
50	27.99000	29.71000	32.36000	34.76000	63.17	67.50	71.42	76.15	79.49	86.66
60	35.53000	37.48000	40.48000	43.19000	74.40	79.08	83.30	88.38	91.95	99.61
70	43.28000	45.44000	48.76000	51.74000	85.53	90.53	95.02	100.4	104.2	112.3
80	51.17000	53.54000	57.15000	60.39000	96.58	101.9	106.6	112.3	116.3	124.8
90	59.20000	61.75000	65.65000	69.13000	107.6	113.1	118.1	124.1	128.3	137.2
100	67.33000	70.06000	74.22000	77.93000	118.5	124.3	129.6	135.8	140.2	149.4

The function tabulated is  $\chi^2_Q$  defined by

$$\int_{\chi^2_Q}^{\infty} f(x) dx = Q; \quad f(x) = \frac{1}{2^{1/2} (\frac{1}{2} \nu - 1)!} x^{1/2 - 1} e^{-x/2} (x > 0)$$

where  $f(x)$  is the probability density of the  $\chi^2$  distribution for  $\nu$  degrees of freedom. Interpolation  $\nu$ -wise for  $\nu > 30$  gives adequate values (but errors up to 5 units in the last figure may occur for the smaller  $\nu$ ). For  $\nu > 100$  the distribution of  $\sqrt{(2 \chi^2)}$  is approximately normal with mean  $\sqrt{(2\nu - 1)}$  and unit variance.

Note:  $0.0^4 = 0.00002$   
 $0.0^3 = 0.0003$   
 $0.0^2 = 0.004$

PERCENTAGE POINTS OF STUDENT'S  $t$ -DISTRIBUTION  $t_Q$

$\nu$	Probability*									$Q$ $2Q$
	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005	
	0.50	0.20	0.10	0.050	0.02	0.010	0.0050	0.002	0.0010	
1	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6	
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60	
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92	
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610	
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869	
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959	
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408	
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041	
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781	
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587	
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437	
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318	
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221	
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140	
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073	
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015	
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965	
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922	
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883	
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850	
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819	
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792	
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767	
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745	
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725	
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707	
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690	
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674	120
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659	$\nu$
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646	4
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551	3
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460	2
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373	1
$\infty$	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291	0

The function tabulated is  $t_Q$  defined by

$$\int_{t_Q}^{\infty} f(t) dt = Q; \quad f(t) = \frac{(\frac{1}{2}\nu - \frac{1}{2})!}{\sqrt{(\nu\pi)(\frac{1}{2}\nu - 1)!}} \cdot \frac{1}{(1 + t^2/\nu)^{(\nu+1)/2}}$$

where  $f(t)$  is the probability density of the  $t$ -distribution.

Interpolation  $\nu$ -wise should be linear in  $120/\nu$  for  $\nu > 30$ .

Use (i) upper row for one tail-tests

(i) lower row for two tail-tests

If  $x$  is a random variable with the  $t$ -probability distribution for  $\nu$  degrees of freedom, the probability that  $x > t_Q$  is  $Q$  and the probability that  $|x| > t_Q$  is  $2Q$ .

The graph shows the form of the distribution for  $\nu = 2$ . The shaded area represents the probability  $Q$ . For large  $\nu$  the distribution approximates to the normal distribution  $N(0,1)$ , shown by the dotted line.

